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Tables of the Symmetric Functions of the Twelfthic.

By W. P. Durfee, Fellow of the Johns Hopkins University.

In the following pages is given in tabular form, 1st the values of the functions (of weight twelve) of the coefficients of the twelfthic $(1, -p_1, p_2, -\dots p_{12})(x, 1)^{12}$ in terms of the symmetric functions of its roots, and 2nd, the values of the symmetric functions of the roots in terms of the coefficients. The notation used is that of Hirsch. For example: $(4^2 21^2) = p_4^2 p_2 p_1^2$; $[4^2 21^2] = \alpha^4 \beta^4 \gamma^2 \delta_{\epsilon}$ where $\alpha, \beta, \gamma, \delta$, &c., are the roots of the given quantic. The value of any function is found from the table by taking the sum of the results obtained by multiplying each coefficient in its row by the expression at the head of the corresponding column.

The symmetry of the results was made use of as a check, the method followed being to calculate at the same time the value of a function, say $[\alpha^a \beta^b \gamma^c]$, and the coefficients of the corresponding function $(\alpha^a \beta^b \gamma^c)$ in the value of each of the root functions. In other words, I calculated simultaneously a row and its symmetric column. An error in a constituent of a row would not affect any other of the row, but would affect those following in its column, while a corresponding error in a column would affect the remainder of that column but no other column. Since the rows and columns agree throughout, the existence of errors is almost impossible. As a final test I made use of the well-known expression for the sum of the homogeneous symmetric functions of the roots:

$$H_n = \left| egin{array}{cccc} p_1 & p_2 & \dots & p_n \\ 1 & p_1 & \dots & p_{n-1} \\ & 1 & p_1 & \dots & p_{n-2} \\ & & \ddots & \ddots & \\ & & 1 & p_1 \end{array} \right|$$

If each coefficient p_1 , p_2 , &c., be made equal to unity, H_n reduces to the sum of its numerical coefficients, which from the resulting form of the determinant is seen to be equal to zero. On summing the coefficients of the second table I found their sum zero as it should be.

Table I. (upper half): Coefficients in Terms of Roots.

 $\begin{bmatrix}1^{1\,2}\end{bmatrix}\ \begin{bmatrix}21^{1\,0}\end{bmatrix}\ \begin{bmatrix}2^{2}18\end{bmatrix}\ \begin{bmatrix}31^{9}\end{bmatrix}\ \begin{bmatrix}2^{3}1^{6}\end{bmatrix}\ \begin{bmatrix}321^{7}\end{bmatrix}\ \begin{bmatrix}41^{8}\end{bmatrix}\ \begin{bmatrix}2^{4}1^{4}\end{bmatrix}\ \begin{bmatrix}32^{2}1^{5}\end{bmatrix}\ \begin{bmatrix}3^{2}1^{6}\end{bmatrix}\ \begin{bmatrix}421^{6}\end{bmatrix}\ \begin{bmatrix}51^{7}\end{bmatrix}\ \begin{bmatrix}2^{5}1^{2}\end{bmatrix}\ \begin{bmatrix}32^{3}1^{3}\end{bmatrix}\ \begin{bmatrix}3^{2}21^{4}\end{bmatrix}$

	L- J		F 7-7	[]	J	[9.02]		[~ _] [.0.0 1]	[0 1]	[101]	[07]		[0.0 1	, .
$(\overline{12})$	1	-													
$(\overline{11} - 1)$	12	1													
$(\overline{10} - 2)$	66	10	1												
(93)	220	45	8		1										
(84)	495	120	28		6			1							
(75)	792	210	56		15			4					1		
(6^2)	924	252	70		20			6					2		
$(\overline{10} - 1^2)$	132	,	2	1											
(921)	660	145	26	9	3	1									
(831)	1980	525	128	36	27	7		4	1						
(82^2)	2970	810	201	72	42	15		6	2	1					
(741)	3960	1170	336	84	93	21		24	5	0			5	1	
(732)	7920	2460	736	252	207	70		52	17	6			10	3	1
(651)	5544	1722	532	126	165	35		52	10	0			17	3	0
(642)	13860	4620	1526	504	498	161		160	50	15			50	15	4
(63^{2})	18480	6300	2128	756	707	252		228	81	30			70	24	9
(5^22)	16632	5670	1932	630	660	210		228	70	20			81	24	6
(543)	27720	9870	3528	1260	1266	455		456	165	60			165	60	22
(4^3)	34650	12600	4620	1680	1710	630		639	240	90			240	93	36
(91^3)	1320	300	54	28	6	- 3	1								
(821^{2})	594 0	1665	418	189	87	38	8	12	- 5	2	1				
(731^{2})	15840	5040	1528	624	432	16 8	28	108	40	12	6		20	7	2
(72^21)	23760	7740	2392	1080	681	302	56	168	72	32	13		30	12	5
(641^{2})	27720	9450	3164	1218	1041	378	56	336	115	30	15		105	34	8
(6321)	55440	19740	6888	3024	2337	1008	168	760	320	132	51		230	93	37
(62^3)	83160	30240	10752	5040	3696	1722	336	1206	552	262	108		360	159	72
(5^21^2)	33264	11592	4004	1512	1380	490	70	480	160	40	20		172	54	12
(5421)	83160	30872	11396	4914	4188	1771	280	1536	635	250	95		565	228	86
(53^21)	110880	42000	15848	7056	59 52	2632	420	2220	976	420	150		820	357	154
(532^{2})	166320	64260	24696	11592	9417	4410	840	3552	1652	776	315		1320	606	279
(4^231)	138600	53550	20720	9240	8040	3570	560	3132	1390	600	210		1225	546	238
(4^22^2)	207900	81900	32270	15120	12720	5950	1120	5022	2340	1090	440		1990	924	424
$(43^{2}2)$	277200	111300	44800	21420	18075	8680	1680	7308	3525	1710	69 0		2960	1434	699
(3^4)	369600	151200	62160	30240	25680	12600	2520	10656	5280	2640	1080		4440	2220	1128
(814)	11880	3420	870	46 8	180	94	33	24	12	6	4	1			
(721^3)	47520	15840	4968	2520	1422	703	204	348	166	78	46	7	60	27	12
(631)	110880	40320	14280	6888	4890	2289	588	1596	722	306	174	21	480	208	84
(62^21^2)	166320	61740	22288	11340	7737	3878	1064	2532	1241	608	331	42	750	354	167
. ,	,		1	l		1					· · · · · · · · · · · · · · · · · · ·	1			

Table I. (upper half continued): Coefficients in Terms of Roots.

	$[42^21^4]$	[4315]	[5215]	[616]	$[2^6]$	[3241] [[322212]	[3313]	$[42^31^2]$	[4321³]	$[4^21^4]$	$[52^21^3]$	[5314]	[6214]	[715]
$(\overline{12})$						}									
$(\overline{11}-1)$															
$(\overline{10}-2)$				Ì											
(93)															
(84)															
(75)															
(6 ²)					1										
$(\overline{10}-1^2)$															
(921)															
(831)															
(822)															
(741)						i									
(732)															
(651)					6	1									
(642)					15	4	1								
(63^2)					20	6	2	1							
(5^22)					30	9	2	0							
(543)					60	22	8	3							
(4^3)					90	36	15	6							
(91³)															
(821^{2})						Ì									
(731^{2})	1					Ì									
(72^21)	2	1													
(641^{2})	4	0			30	9	2	0	1						
(6321)	14	5			60	22	8	3	3	1					
(62^3)	30	16			90	36	15	6	6	3	1				
(5^21^2)	6	0			6 0	20	4	0	2	0	0		ļ		
(5421)	32	10			210	83	30	9	11	3	0				
(53^21)	53	20			300	128	54	24	18	7	0				
(532^{2})	114	55			480	216	96		39	18	4	:			
(4^231)	80	30			480	216	96	42	31	12	0				
(4^22^2)	172	80			795	368	168	72	68	30	1	1			
$(43^{2}2)$	284	140			1200	584	286			l .	l .	į.			
(3^4)	468	240			1860	936	480	256	204	108	24	:			
(814)															
(721^{3})	7	3	1												
(631^{3})	47	15	5		120	48	18	6		l	i		1		
$(62^{2}1^{2})$	90	43	11		180	78	34	15	18	8	2	2	1	<u> </u>	

Table I. (upper half continued): Coefficients in Terms of Roots.

	$[3^22^3]$	[3321]	[424]	$[432^21]$	$[43^21^2]$	$[4^221^2]$	[5231]	$[3^4]$	$[43^{2}2]$	$[4^22^2]$	[4231]	$[53^21]$	[5421]	$[4^{3}]$	[543]	$[5^{2}2]$
$(\overline{12})$																
$(\overline{11}-1)$																
$(\overline{10}-2)$																
(93)																
(84)									!							
(75)																
(6 ²)																
$(\overline{10}-1^2)$																
(921)		į														
(831)																
(822)																
(741)																
(732)																
(651)																
(642)																
(63 ²)																
(5 ² 2)	1															
(543)	3	1														
(4 ³)	6	3						1								
(91³)																
(821 ²)																
(731 ²)																
(7221)																
(641 ²)																
(6321)																
(62 ³)	1															
(5^21^2)	2	0	1													
(5421)	11	3	4													
(5321)	18	7	6		1											
(532^2)	31	12	12		2	1										
(4 ² 31)	39	18	12		2	0		4								
(4^22^2)	68	30	28	1	4	2		6	1		1 1					
(4322)	117	58	48	1	12	5		12		2	1 1					
(34)	204	108	90	48	28	12		24	12	6	4			1		
(814)																
(7213)]										1
(631 ³)																
(62^21^2)																

Table I. (lower half): Coefficients in Terms of Roots.

	[112]	[2110]	$[2^218]$	[319]	[2816]	[3217]	[41 ⁸]	[2414]	[32 ² 1 ⁵]	$[3^21^6]$	[421 ⁶]	[517]	[2 ⁵ 1 ²]	[32313]	[32214]
(541³)	166320	63000	23604	11088	8766	3983	966	3240	1420	570	320	35	1200	507	192
(5321 ²)	332640	131040	51128	25704	19722	9793	2548	7500	3670	1762	926	105	2800	1344	632
$(52^{8}1)$	498960	200340	79632	41580	31221	16212	4536	12012	6177	3192	1713	210	4500	2277	1153
(4^221^2)	415800	166950	66780	33390	26640	13160	3360	10620	5180	2460	1280	140	4245	2046	954
(43^21^2)	554400	226800	92680	47040	37860	19110	4900	15468	7780	3840	1950	210	6320	3174	1570
(432^21)	831600	346500	144200	75600	59925	31360	8680	24864	12980	6760	3575	420	10300	5361	2779
(42^4)	1247400	529200	224280	120960	94860	51240	15120	40002	21600	11760	6420	840	16800	9054	4900
(3321)	1108800	470400	199920	105840	85140	45150	12600	36336	19320	10260	5400	630	15540	8289	4428
(3^22^3)	1663200	718200	310800	168840	134760	73500	21840	58536	32040	17660	9630	1260	25470	13986	7728
(71 ⁵)	95040	32400	10320	5760	2970	1620	600	720	380	200	135	36	120	60	30
(621^4)	332640	126000	46200	25200	16200	8680	2940	5316	2780	1440	910	196	1560	786	392
(531^4)	665280	267120	105840	56448	41310	21644	6888	15840	8132	4056	2489	476	5940	2976	1442
$(52^{2}1^{3})$	997920	408240	164808	90720	65412	35658	11844	25380	13656	7308	4446	882	9540	5037	2640
(4^21^4)	831600	340200	138180	73080	55800	28980	9030	22464	11440	5620	3420	630	9060	4524	2160
(4321³)	1663200	705600	298200	163800	125550	68495	22260	52668	28530	15270	9110	1715	22020	11841	6284
(42^31^2)	2494800	1077300	463680	260820	198765	111510	37800	84780	47385	26460	15975	3150	35940	19986	11085
(3313)	2217600	957600	413280	228480	178380	98280	31920	77040	42360	23040	13590	2520	33300	18300	9972
$(3^22^21^2)$	3326400	1461600	642320	362880	282360	159460	54040	124176	70120	39520	23720	4620	54640	30858	17404
(32^41)	4989600	2230200	997920	574560	446940	257880	90720	200328	115800	67200	40980	8400	89850	52020	30220
(2^6)	7484400	3402000	1549800	907200	707400	415800	151200	323460	190800	113400	70200	15120	148140	87660	52200
(61 ⁶)	665280	257040	95760	55440	33930	19320	7560	11160	6210	3440	2355	672	3240	1740	930
(5215)	1995840	831600	341040	196560	137070	78120	29400	53640	30130	16800	11085	2856	20220	11130	6070
(4315)	3326400	1436400	616560	352800	263070	149100	54600	111600	62600	34600	22425	5460	47100	26130	14250
(42^21^4)	4989600	2192400	958440	55944 0	416520	241920	91140	179748	103780	59640	38670	9660	76920	44088	25120
(3^221^4)	6652800	2973600	1327200	776160	591660	344960	129360	263520	153220	88560	56990	14000	117300	68040	39244
$(32^{3}1^{3})$	9979200	4536000	2061360	1224720	936540	556290	214200	425376	252600	149940	97200	24570	193140	114657	68040
(2^51^2)	14968800	6917400	3200400	1927800	1482300	894600	352800	687240	415500	252000	164700	42840	318930	193140	117300
(51^7)	3991680	1693440	705600	423360	287280	170520	70560	113400	66360	38640	26880	8232	42840	24570	14000
(421^6)	9979200	4460400	1980720	1194480	872910	523320	214200	381240	226950	134400	91725	26880	164700	97200	56990
(3^21^6)	13305600	6048000	2741760	1653120	1239840	744240	302400	559440	334320	198400	134400	38640	252000	149940	88560
(32^21^5)	19958400	9223200	4257120	2600640	1962540	1197000	495600	903600	550240	334320	226950	66360	415500	252600	153220
(2^41^4)	29937600	14061600	6607440	4082400	3106080	1920240	808920	1460736	903600	55 944 0	381240	113400	687240	425376	263520
(41 ⁸)	19958400	9072000	4092480	2540160	182 9 520	1128960	493920	808920	495600	302400	214200	70560	352800	214200	129360
(321^7)	39916800	18748800	8789760	5503680	4112640	2569560	1128960	1920240	1197000	744240	523320	170520	894600	556290	344960
(2°16)	59875200	28576800	13638240	8618400	6508620	4112640	1829520	3106080	196254 0	1239840	872910	287280	1482300	936540	591660
(31^9)	79833600		18144000	11612160	8618400	5503680	2540160	4082400	2600640	1653120	1194480	423360	1927800		776160
(2218)	119750400			18144000	13638240	8789760	4092480	6607440	4257120	2741760	1980720	705600	3200400		
(2110)	239500800			í	i							1	6917400		1
(112)	479001600	239500800	119750400	79833600	59875200	39916 800	1 99 58400	2 99 37600	19958400	13305600	9979200	39916 80	14968800	9979200	6652800

Table I. (lower half continued): Coefficients in Terms of Roots.

	$[42^21^4]$	[4315]	[521 ⁵]	[61 ⁶]	[26]	[3241]	$[3^22^21^2]$	[3313]	[42312]	[4321³]	[4 ² 1 ⁴] [[52 ² 1 ³]	[531 ⁴]	[6214]	[715]
(541^3)	106	30	10		450	184	66	18	36	9	0	3	0		
(5321 ²)	327	145	35		1020	476	216	96	110	47	8	11	4		
(5231)	618	321	75		1620	798	390	189	207	103	30	24	13		
(4^221^2)	488	210	50		1710	817	376	162	189	78	12	18	6		
(43^21^2)	781	360	. 80		2580	1296	644	318	314	147	24	31	12		
(432^21)	1466	755	170		4260	2210	1140	582	599	305	80	68	34		
(42^4)	2694	1500	360		7020	3768	2026	1080	1116	610	201	150	88		
(3^321)	2328	1230	270		6660	3564	1914	1038	1008	540	144	117	60		
(3^22^3)	4248	2390	570		11100	6114	3386	1884	1875	1056	344	258	150		
(71 ⁵)	20	10	5	1				:							
(621^{4})	245	120	50	6	360	168	78	36	48	22	6	9	4	1	
(531^4)	872	390	155	15	2160	1048	488	216	291	124	24	48	16	4	
(52^21^3)	1593	828	306	30	3420	1752	886	444	528	264	78	96	48	9	
(4^21^4)	1294	560	220	20	3690	1812	844	360	498	204	36	78	24	6	
(4321^3)	3711	1905	675	60	9180	4900	2578	1332	1506	763	204	264	124	22	
(42^31^2)	6678	3705	1305	120	15120	8352	4592	2505	2754	1506	498	528	291	48	
(3^31^3)	5820	3060	1050	90	14400	7920	4332	2364	2505	1332	360	444	216	36	
$(3^22^21^2)$	10428	5840	2020	180	24060	13592	7672	4332	4592	2578	844	886	488	78	
(32^41)	18504	10860	3840	360	40320	23376	13592	7920	8352	4900	1812	1752	1048	168	
(2^6)	32580	19800	7200	720	67950	40320	24060	14400	15120	9180	3690	3420	2160	360	
(61^6)	630	335	171	37	720	360	180	90	120	60	20	30	15	6	1
(521^{5})	3970	2135	9 85	171	7200	3840	2020	1050	1305	675	220	306	155	50	5
(431^{5})	9140	4800	2135	335	19800	10860	5840	3060	3705	1905	560	828	390	120	10
(42^21^4)	16201	9140	3970	630	32580	18504	10428	5820	6678	3711	1294	1593	872	245	20
(3^221^4)	25120	14250	6070	930	52200	30220	17404	9972	11085	6284	2160	2640	1442		30
(32^31^3)	44088	26130	11130	1740	87660	52020	30858	18300	19986	11841	4524	5037	2976	786	60
$(2^{5}1^{2})$	76920	47100	20220	3240	148140	49850	54640	33300	35940	22020	9060	9540	5940	1560	120
(51^7)	9660	5460	2856	672	15120	8400	4620	2520	3150	1715	630	882			36
(421^6)	38670	22425	11085	2355	70200	40980	23720	13590	15975	9110	3420				135
(3^21^6)	59640	34600	16800	3440	1	ł	39520	23040				7308 	4056		200
(32^21^5)	103780	62600	30130	6210	190800			42360							380
(2^41^4)	179748	111600	530640		1									1	720
(41^8)	91140	54600	29400	i	151200									!	600
(321^7)	241920	149100		19320	415800	257880	159460								1620
(2^31^6)	416520	263070	137070			1			i					1	2970
(31^9)	559440	352800			907200				i				56448		5760
(2^21^8)	1	616560			1549800	ł				1				1	10320
(21^{10})	2192400			1	i	1	1		1	705600			i		32400
(1^{12})	4989600	3326400	1995840	665280	7484400	4989600	3326400	2217600	2494800	1663200	831600	997920 	665280	332640	95040

Table I. (lower half continued): Coefficients in Terms of Roots.

	[3223]	[3321]	[42 ⁴] [432²1] [[43 ² 1 ²] [4 ² 21 ²]	[5 2 ³1] [[5321²]	[5 41 ³] [62 ² 1 ²]	[631³]	[721 ³]	[814]	[34]	[4322]	[4222]
(541³)	24	6	13	3	0	0	1									
(5321 ²)	68	27	34	13	5	2	3	1							į	-
(5231)	117	51	60	27	12	7	6	3	1		Ì		}			Ì
(4^221^2)	153	66	76	31	10	4	7	2	0					12	5	2
(43^21^2)	264	132	126	60	29	10	12	5	o					28	12	4
(43221)	467	237	244	123	60	31	27	13	3	Ì				48	24	12
(42^4)	828	438	456	244	126	76	60	34	13					90	48	28
(3^321)	828	451	438	237	132	66	51	27	6					108	58	30
(3^22^3)	1486	828	828	467	264	153	117	6 8	24					204	117	68
(71 ⁵)														i		
(621^4)		Ì			Ì											ļ
(531^4)	150	60	88	34	12	6	13	4	0	1						1
(52^21^3)	258	117	150	6 8	31	18	24	11	3	2	1					
(4^21^4)	344	144	201	80	24	12	30	8	0	2	0			24	12	6
(4321^3)	1056	540	610	305	147	78	103	47	9	8	3			108	58	30
(42^31^2)	1875	1008	1116	599	314	189	207	110	36	18	10			204	117	6 8
(3^31^3)	1884	1038	1080	582	318	162	189	96	18	15	6			256	141	72
$(3^22^21^2)$	3386	1914	2026	1140	644	370	390	216	66	34	18			480	286	168
(32^41)	6114	3564	3768	2210	1296	817	79 8	476	184	78	48			936	584	368
(2^6)	11100	6660	7020	4260	2580	1710	1620	1020	450	180	120			1860	1200	795
(61^6)																
(5215)	570	270	360	170	80	50	75	35	10	11	5	1				
(4315)	2390	1230	1500	755	360	210	321	145	30	43	15	3		240	140	80
(42^21^4)	4248	2328	2694	1466	781	488	618	327	106	90	47	7		468	284	172
(3^221^4)	7728	4428	4900	2779	1570	954	1153	632	192	167	84	12		1128	699	420
(32^31^3)	13986	8289	9054	5361	3174	2046	2277	1344	507	354	208	27		2220	1434	924
(2^51^2)	25470	15540	16800	10300	6320	4245	4500	2800	1200	750	480	60		4440	2960	1990
(51^7)	1260	630	840	420	210	140	210	105	35	42	21	7	1			
(421 ⁶)	9630	5400	6420	3575	1950	1280	1713	926	320	331	174	46	4	1080	690	440
(3^21^6)	17660	10260	11760	6760	3840	2460	3192	1762	570	608	306	78	6	2640	1710	1090
(32^21^5)	32040	19320	21600	12980	7780	5180	6177	3670	1420	1241	722	166	12	5280	3525	2340
(2^41^4)	58536	36336	40002			10620	12012	7500	3240	2532	1596	348	24	10656	7398	5022
(41^8)	21840	12600	15120	8680	4900	3360	4536	2548	966	1064	588	204	33	2520	1680	1120
(3217)	73500			1					3983	3878	2289	703	94	l	Į.	
(2^31^6)	134760								8766	7737	4890	1422	180	ì	1	12720
(31°)	168840			l			41580				[2520	468	l	ŀ	
(2^21^8)	310800			144200	i		79632					4968			44800	
(2110)	1	470400	1	ì	226800		1	1				1	1	1	i	81900
(1^{12})	1663200	1108800	1247400	831600	554400	415800	498960	332640	166320	166320	110880	47520	11880	369600	277200	207900

Table I. (lower half continued): Coefficients in Terms of Roots.

TABLE I. (lower half continued): Coefficients in Terms of Roots.

	[642]	[651]	[732]	[741]	[822]	[831]	[921]	$[10-1^2]$	[6 ²]	[75]	[84]	[93]	[10-2]	[11-1]	[12]
(541^3)						İ				1					
(5321 ²)														İ	1
(52^31)														1	1
(4^221^2)				1										}	
(43^21^2)			ļ									į			
(432^21)														1	
(42^{4})											Ì			Ì	1
(3^321)															1
(3^22^3)		İ													
(71^{5})															
(621^4)		1			1					}					1
(581^{4})				1		ĺ									1
(52^21^3)															
(4^21^4)															
(43213)						ļ									1
$(42^31^{\frac{1}{2}})$											ĺ				
(2 ³ 1 ³)			}												1
$(8^22^21^2)$	1		Ì												
(32^41)	4	1	,												
(2^6)	15	6	Ì						1						
(61^6)					Ì										
(5215)															
(431^5)				1											
(42^21^4)				į					·						
(3^221^4)	4	0	1				1								
(32^31^3)	15	3	3	1	Ì										
(2^51^2)	50	17	10	5					2	1					
(51^7)															I
(421^6)															
(3^21^6)	15	0	6	o	1										ļ
(32^21^5)	50	10	17	5	2	1									
(2^41^4)	160	52	52	24	6	4			6	4	1				
(41 ⁸)															
(3217)	161	35	70	21	15	7	1	l							
(2^31^6)	498	165	207	98	42	27		3	20	15	6	1	ı l		
(31°)	504	126	252	84	72	36) 1							
(2^21^8)	1526	532	736	336	201	128	26	1		56	28	8	3 1		
(21 ¹⁰)	4620	1722	2460	1170	810	525	148	Į.	l	210	[l	Į.	1	
(1 ¹²)	13860	5544	7920	3960	2970	1980	660	132	924	792	495	220	66	12	1

TABLE II. (upper half): Roots in Terms of Coefficients.

	(12)	(11-1)	(10-2)	(93)	(84)	(75)	(6^2)	(1012	(921)	(831)	(82^2)	(741)	(732)	(651)	(642)	
[112]	1															
[2110]	- 12	1														
$[2^21^8]$	+ 54	l	1													
[31 ⁹]	+ 12								1							
$[2^31^6]$	1	+ 35	- 8	1												
[3217]	1	1	+ 16	- 3				_ ,	0 1							
[418]	_ 12	+ 1	+ 2	+ 3				- :	1 - 3							
$[2^41^4]$	+ 105	— 50	+ 20	- 6	1											1
$[32^{2}1^{5}]$	+ 252	- 87	— 32	+ 18	- 4			+ 2'	7 - 7	1						
$[3^21^6]$	+ 42	— 9	- 17	+ 3	+ 2			+ 9	– 1	- 2	1					
[421]	+ 84	— 18	14	- 21	+ 4			+ 1	+ 23	- 1	- 2					
[517]	+ 12	— 1	- 2	- 3	- 4			+ :	1 + 3	+ 4	+ 2					
$[2^51^2]$	- 36	+ 25	- 16	+ 9	- 4	1								,		
$[32^31^3]$	240	+ 130	0	- 21	+ 16	- 5		- 30	+ 14	– 5	0	1				
$[3^221^4]$	— 180	+ 70	+ 70	— 36	- 4	+ 5		- 48	+ 13	+ 10	- 6	3	1			
$[42^21^4]$	— 180	+ 70	+ 20	+ 45	— 20	+ 5		- 20	– 54	+ 5	+ 12	- 1	_ 2			
[4315]			+ 32	1	}	— 5		- 17	-22	+ 5	0	+ 5	- 1			
[5215]	— 72	+ 17	+ 12	+ 18	+ 24	5		- 7	-20	— 27	- 12	+ 1	+ 5			
$[61^6]$	— 12	+ 1	+ 2	+ 3	+ 4	+ 5		<u> </u>	- 3	- 4	- 2	— 5	— 5			
[26]	+ 2	- 2	+ 2	- 2	+ 2	- 2	1									
[3241]			+ 20	- 6	1	+ 10		1	1 1	+ 5	0	— 3	0	1		
$[3^22^21^2]$			— 45			- 5		ļ	H — 30		· 1	+ 9		- 4	1	
$[3^31^3]$	+ 40		- 30	l	1	- 5		1	1 1	- 11		+ 3		+ 2	_	
$[42^31^2]$	i i	1	+ 10	- 1	- 1	- 15			39		- 18					
[43213]			100		- 1				3 + 73	1	. 1	— 17 5	+ 7			
$[4^21^4]$	+ 30	- 1	— 15		1				3 + 23		ſ	$- 5 \\ - 4$				1
$[52^21^3]$ $[531^4]$	+ 120		- 10	1				ĺ	+ 37		i i			$+ 1 \\ - 6$	· .	1
[621 ⁴]	+ 60	- 1	- 30 - 10		-20 -20			1	3 + 19 3 + 17		1				- 6	
[71 ⁵]	+ 12	1	— 10 — 2	_ 3	-20 -4	— 20 — 5		1 '	1 1		+ 2			+ 6	+ 6	
$[3^22^3]$			1	1				, .	1 .							1
$[3^{3}21]$																
[424]									2 4							
[432 ² 1]	— 144	+ 111	+ 24	+ 9	- 16	+ 4	(_ 5	1 - 36	+ 16	+ 24	+ 3	— 19	- 1	_ 8	
[43 ² 1 ²]	— 72	+ 39	+ 62	_ 9	— 40	+ 2	+ 18	3 - 39	-20	+ 34	- 6	+ 15	- 4	_ 20	+ 2	
									4 — 62							
[5231]																
١ -		<u> </u>	- 1		· - 1			 	1	l					l	1

TABLE II. (upper half continued): Roots in Terms of Coefficients.

,	(63 ²) (5	22)	(543)	(91 ³	3) ((821²)	(731 ²)	(7221)	(641 ²	(632	31)	(623)	(5 ² 1 ²)	(5421)	(53°1)	(5322)
[1 ¹²]																	
[2110]											-						
$[2^21^8]$																	
[31°]																	
$[2^31^6]$																	
[3217]																	
$[41^{8}]$																	
$[2^41^4]$																	
$[32^21^5]$																	
$[3^21^6]$																	
$[421^{6}]$					-	8	1										
[51 ⁷]			1			1	— 4										
$[2^{5}1^{2}]$																	
$[32^31^3]$																	
$[3^221^4]$																	
$[42^21^4]$					+	20	- e	1									
$[431^{5}]$					+	8	— 1	<u> </u>		1							
$[521^5]$					+	7	+ 27	- 1	-	3							
[61 ⁶]					+	1	+ 4	+ 5	+	5							
$[2^{6}]$																	
$[32^41]$																	
$[3^2 2^2 1^2]$																	
$[3^{3}1^{3}]$		1										-					
$[42^31^2]$		0				16	+ 8	- 4		0	1						
$[4321^3]$	-	3			-	32	+ 11	+ 8	-	5 —	3	1					
$[4^21^4]$	+	3			-	8	+ 1	+ 2	-	1 +	3 —	3	1				
$[52^21^3]$	+	3			-	14	51	+ 4	+ 1	5 —	1 —	3	0	1			
$[531^4]$	-	3	.		-	7	— 27	+ 8	+ :	8 +	2 +	4	_ 2	,			
$[621^{4}]$	-	3			-	6	— 23	- 29	- 2	7 +	1 +	7	+ 2	;			
[715]	+	3			-	1	- 4	— 5	-	5 —	6 —	12	— 2	1			
$[3^22^3]$		0	1									1					
$[3^321]$	-	3 —	3		1 .												
$[42^{4}]$		0 —	2		0 +	2	_ 2	+ 2	(0 —	2	0	0	1			
$[432^21]$	+	9 +	6		1	- 1	- 17	0	+ +	5 +	6 —	3	0	- 4	1		
$[43^21^2]$		0 +	3		1 +	- 1		1	+	1	3 —	1		+ %	_ 2	1	
$[4^221^2]$		9 —	7	+	5 +	24	- ⋅ 10	- 6	+ 4		9 +	11	- 4	+ 4	- 1	_ 2	1
$[52^{3}1]$	-	9 +	2	+ :	3 +	7	+ 23	- 2	- 1	5 +	2 +	9	0	1	_ 3	0	0

Table II. (upper half continued): Roots in Terms of Coefficients.

	(81	⁴)	(721³)	(6313)	(62212) (5 4 1³)	(71^5)	(621 ⁴)	(52213	(4^21^{4})	(61^6)	(5215)	(4315)	(42^21^4)
$[1^{12}]$					1									
[21 ¹⁰]														
$[2^{2}1^{8}]$														
[31°]														
[2316]														
[8217]														
[418]							:							
[2414]														
$[82^21^5]$														
$[3^21^6]$														
[421 ⁶]														
[517]		1												
$[2^{5}1^{2}]$														
$[32^{3}1^{3}]$														
$[3^221^4]$														
$[42^21^4]$														
[431 ⁵]														
[5215]		7	1											
$[61^6]$		1	<u> </u>				1							
[26]														
$[32^41]$														
[322212]														
$[3^31^3]$														
$[42^31^2]$														
[48213]														
$[4^21^4]$														
$[52^21^3]$	+			1	1									
[5314]	+	7		1	ł									
[6214]	+		+ 29	1	l — 4		- 6	l						
[715]	+	1	+ {	1 + 6	+ 9		- 1	- 6			1			
[3223]														
[3321]														
[424]														
$[432^21]$														
[43 ² 1 ²]														
[4 ² 21 ²]		ry	٠ ــــــــــــــــــــــــــــــــــــ		3 0	1								
$[52^{3}1]$		-7	+ (š	1 0	1							<u> </u>	

Table II. (lower half): Roots in Terms of Coefficients.

	(12	3)	(11-	-1)	(10	-2)	(98	3)	(84	l)	(7	5)	(6	2)	(101	L²)	(92	21)	(83	1)	(82	²)	(74	1)	(73	32)	(65	1)	(64	2)	(63	2)
[5321 ²]		144	+	78	+	64	+	9	+	48		31		0		48		43		63	_	32	+	21	+	39	+	7		16	+	9
$[541^3]$		48	+	15	+	28	+	39	+	16		22		12		15		43		22	_	12	+	11	+	3	+	19	+	4	_	3
$[62^{2}1^{2}]$		72	+	39	+	2	+	18	+	24	+	37	_	18		9		23	_	33		10		42		27	+	2	+	22	+	9
[631 ³]	_	4 8	+	15	+	28	+	12	+	16	+	13		12		15		16		19		12		24	_	26	+	14	+	4	+	6
[7213]	_	48	+	15	+	8	+	12	+	16	+	13	+	24		5		14	_	19	_	8	_	24		20		28		24		12
[814]	_	12	+	. 1	+	2	+	3	+	4	+	12	+	6		1		3		4	_	2		5		5		13		6		3
$[3^4]$	+	3	_	3	_	3	+	6	_	3	_	3	+	3	+	3	_	3	_	3	+	3	+	6		3	_	3	_	3	+	3
$[43^{2}2]$	+	36		36	_	16	+	18		4	_	1		0	+	26	_	2		14		0	+	5	+	5	+	1	+	8	_	9
$[4^22^2]$	+	18	_	18	+	2		18	+	22	_	18	+	9	+	8	+	16	_	4		14		4	+	16		0	-	12		0
$[4^231]$	+	36		25	_	36	_	9	+	44	—	1		18	+	25	+	34	_	34	_	4		26	+	10	+	26		8	+	9
$[532^{2}]$	+	36		36	+	4		9	_	20	+	34		18	+	16	+	5	+	29	+	4	_	14	—	23	+	2	+	16	+	9
$[53^21]$	+	36		25	_	36	+	18		4	_	1		0	+	25	+	7		1	+	20	_	6	-	17	+	5	+	4		9
[5421]	+	72	_	50	_	32	_	45	-	8	+	33		0	+	30	+	73	+	31		0	_	19		28	_	25	+	16	+	9
$[5^21^2]$	+	18		7	-	13	-	18	_	18	+	17	+	9	+	7	+	20	+	25	+	13		10	_	4		16	_	5		0
$[62^{3}]$	+	12		12	+	8		12	+	4	_	12	+	6	+	2	+	4	+	8		0	+	8	+	4		. 0		12		0
[6321]	+	72	-	50		32	+	9	-	4 0	_	37	+	36	+	30	+	19	+	33	+	16	+	55	+	30		27		24	_	27
$[641^{2}]$	+	36	_	14		26		36		4	_	1	+	18	+	14	+	40	+	18	+	10	+	11	+	27		27	_	18		0
$[72^{2}1]$	+	36		25	+	4	_	9	_	20		1	—	18	+	5	+	12	+	18	+	4	+	24	+	12	+	26	+	16	+	9
$[731^{2}]$	+	36		14	-	26	_	9	_	4	_	1	_	18	+	14	+	13	+	15	+	10	+	11	+	21	+	15	+	30	+	9
$[821^2]$	+	36	-	14	-	6		9		4	-	36	_	18	+	4	+	11	+	15	+	6	+	18	+	.15	+	50	+	10	+	9
[91 ³]	+	12	_	1	-	2		3	_	12	_	12		6	+	1	+	3	+	4	+	2	+	13	+	5	+	13	+	14	+	3
$[4^{8}]$	-	4	+	4	+	4	+	4	_	12	+	4	+	2		4	_	8	+	8	+	4	+	8		8	_	8	+	8		4
[543]	-	24	+	24	+	24		3	_	8	_	11	+	12	-	24	_	21	+	11		8	+	19	+	14	_	13	-	16	+	3
$[5^{2}2]$	-	12	+	12	+	2	+	12	+	12	_	23	+	6	_	7	_					1		- 1	+	21	+	11	_	14		12
$[63^{2}]$	_	12	+	12	+	12	-	15	+			- 1			-	- 1		- 1				- 1				- 1	-		+	- 1		6
[642]	-	24			+										-	1		- 1				- 1		- 1		- 1		- 1		- 1		12
[651]	-	24	+	13	+	24	+	24	+	24	-	11	-	24	-	13	-	37		37	_	24	_	2	_	13	+	40	+	24	+	12
[732]	-	24	+	24	+	4	-	3	+	24	-	11	+	12	-	14	_	1	-	21	_	ı		- 1		- 1		13	_	28	+	3
[741]	1	24								8	-	11	+	12	-	13		37	_	5	-	8	+	2	_	- 1					_	- 1
$[82^2]$	-	12	+	12	-	8	+	12	-	4	+	12	+	6	—	2	— <u>,</u>	4 ت	_	8		0	_	8	_	4		24	+	12	_	12
[831]	I .		i i		1						1	- 1			-	- 1		- 1						1		1				- 1		
[921]															_																	
$[101^{2}]$															<u> </u>																	
$[6^2]$	1		I		i		ı				i	- 1			+	1		- 1						- 1		- 1		1		- 1		
[75]	1				t		j				i				+			- 1												- 1		
[84]															+																	
[93]															+																	
[102]															+																	
[111]															+																	
[12]		12	+	12	+	12	+	12	+	12	+	12	+	6	-	12	_	24		24	<u> </u>	12	_	24		24	_	24	_	24	_	12

TABLE II. (lower half continued): Roots in Terms of Coefficients.

[5321²] **—** 15 | **—** 23 | **—** 3 + + 3 + 21 + 75- 11 $[541^3]$ $[62^21^2]$ 33 ---3 - 25 8 + + [631⁸] 3 --27 28 -11 [721⁸] 22 + 29 + 53 + $[81^{4}]$ 5 + 5 + 6 + 12 +2 + $[8^4]$ + [43²2] 9 + 8 + 8 + 2 7 + 0 + $[4^22^2]$ 2 + $[4^231]$ + 16 +9 + 169 + 10 + 10 10 1 0 $[532^{2}]$ 11 **— 21 +** + 19 — 2 -11 4+ 1 3 + 2 + 2 $[53^21]$ + 2 + 14 -5 10 + 11 7 - 24 + 118 + 8 0 + [5421]7 0 + 103 + 101 -39 + 1610 30 18 + 10 $[5^21^2]$ 3 2 + + 9 + 1 + 0 $[62^{3}]$ - 10 [6321] + 17 -4 + 8 - 12 - 41 - 49 - 42 +5 +56 8 + 1 10 + $[641^{2}]$ -22|-25|-29|+17|+5|+2 +9 + 4 — 14 2 + $[72^21]$ 6|+5|-17|-23|-16|-29|-42|-4 + 18 +8 + 193 + $[731^{2}]$ 19 - 22 -23|-25|-49|-10|+16 +11 十 -17|-22|-41|24 - 21 $[821^2]$ + 21 +13 15 — 19 6 - 27 -39 8 [91³]7 + 15 +5 5 - 14 - 12 -7 30 1 $[4^{3}]$ 4 8 4+ 6 8 8 + [543]15 + 13 - 22 6 + 5 + 1 11 9 $[5^22]$ 21 19 4 + 17 + $[63^2]$ 0 - 27 4 + 9 + + 5 8|+14|+10|+30|+16|-18|-24|-12|+ 16 +[642][651]8 + 13 + 50 + 15 + 26 - 27- 27 0 - 16 25 +4 5 + 15 + 21 + 12 + 27 + 30 +-28 - 17 - 23 + 10 + 16 +[732]8 + 8 + 13 + 18 + 11 + 24 + 11 + 55 +8 -- 10 -- 19 6 - 14 [741]+ 11 + 19 + $[82^{2}]$ 0 + 130 + 20 +4 + 6 + 10 +4 + 10 + 16[831] 8 + 4 + 15 + 15 + 18 + 18 + 33 +8 + 25 + 3129 — 73 +[921] + 3 + 11 + 13 + 12 + 40 + 19 +4 + 20 +7 + 8 + 267 + 30 + 25 + 16 + 25 + $[101^2]$ + 1 + 4 + 14 +5 + 14 + 30 +2 + 6|-18|-18|-18|+18|+36|+19 — $[6^2]$ 2 ---6|+**—** 12| **—** 36| **—** 1 - 37 - 12 + 17 + 33 - 1 +34 [75] 1 --1 -4 - 4 - 20 - 4 - 40 + 4 - 18 -8 4 - 20 + 44 + 22 -[84] 8 - 12 - 129|-36|+9|-12|-18|-45|+18|-9 — [93] |4| - |2| - |6| - |26| + |4| - |26| - |32| + |8| - |13| - |32| - |36| + [102] 2 + 24 +|4| - |1| - |14| - |14| - |25| - |14| - |50| - |12| - |7| - |50| - |25| - |36| - |25| - |18| - |36|[111] |4| + |12| + |36| + |36| + |36| + |36| + |72| + |12| + |18| + |72| + |36| + |36| + |36| + |36| + |36| + |36|[12]12 — 24 —

Table II. (lower half continued): Roots in Terms of Coefficients.

 $(3^4) \quad (81^4) \quad (721^3) \quad (631^3) \quad (632^1)^2 \quad (541^3) \quad (5321^2) \quad (52^31) \quad (4^221^2) \quad (432^21) \quad (432^21) \quad (42^4) \quad (3^321) \quad (3^22^3) \quad (71^5) \quad (3^4) \quad (3$ $[5321^{2}]$ $[541^3]$ 3 $[62^21^2]$ + [631³] 29 + 2 + + [721³] 24 29 + $[81^{4}]$ + 5 21 $[3^4]$ $[43^{2}2]$ $[4^22^2]$ $[4^231]$ $[532^{2}]$ [53²1] [5421] $[5^21^2]$ $[62^3]$ [6321] 12 53 11 25 11 + $[641^{2}]$ 29 + 6 $[72^{2}1]$ 33 - 23 5 22 28 1 15 $[731^{2}]$ 2 5 27 42 15 $[821^2]$ 32 + 26 75 +23 4 $[91^3]$ 9 + 15 + 21 +7 + 24 + 12 +1 + $[4^3]$ [543] 7 3 + $[5^22]$ + $[63^{2}]$ 3 + 12 [642]2 22 16 [651]28 14 19 20 [732] 20 26 27 3 + 39 + 19 [741] + 24 24 42 + 11 + 21 +3 15 + 15 $[82^{2}]$ 12 - 10 **— 12 — 32** 16 24 +[831] 22 **— 63** -26 + 16 + 34 +16 4 19 19 33 [921] **— 62 — 20** 3 **— 43 — 43** - 16 36 **— 23** $[101^2]$ - 48 24 39 51 27 15 $[6^2]$ 24 12 **— 18** 12 + 12 18 [75] 22 **— 31 — 22 +** 2 13 +37 — [84] 16 + 24 + 16 + 48 + 16 - 40 - 40- 16 [93] 12 + 18 +39 + 9 + 21 +12 12 45 [102] 2 + 28 + 2|+28|+64|-12|+32|+62|+24|8 + 28 -[111] 1 + 15 + 15 + 39 + 15 + 78 + 37 + 39 + 39 + 111 +12 + 37 +[12]48|-144|-48|-72|-72|-144|48 **— 72**

Table II. (lower half continued): Roots in Terms of Coefficients.

	(621	¹) (531	⁴) (52²	1³)	(4 2]	L4)(432	1³)((42³	1²)	(33)	L³)(3 ² 2	² 1 ²	(32	41)	(2^6)	(61	⁶) (521	⁵)	(431	5) (42 21	l ⁴) ((3 ² 2	14)
[5321 ²]																				T									
[541 ³]																													
$[62^21^2]$	_	4		1																				}			-		
[631³]	_	1	_	2		1																							
[7213]	+	29	_	1	_	5														-	5		1						
[814]	+	6	+	7	+	14														-	1	_	7						Ì
$[3^4]$																													
$[43^{2}2]$																													
$[4^22^2]$																		}											
$[4^231]$																													-
$[532^{2}]$										ĺ																			
$[53^21]$																													
[5421]		j																											
$[5^21^2]$																											- 1		
[623]	+	2	_	2		0		1																					
[6321]	+	7	+	4		3		3		1																			
[6412]	+	1	+	2		1	+	3	_	3		1								١.	-		9		4				}
$[72^{2}1]$ $[731^{2}]$	_		+			15		1		5		. 0								+	5 *		3		1				Ì
[821 ²]		23	+		+		+	2		11		9								+	5	1	1 27		2		l G		
[913]	_	6		7	_	51 14		1 8		32		16								+	4	++	7	}		_	20		
[4 ³]				•		14				32		10									1	1	•	T		+	20		
[543]										Ì																			
$[5^{2}2]$						ļ																							
[63 ²]	_	3		3	+	3	+	3	_	3		0		1															
[642]	_	- 1			+	2		3		4	_	2	1	2		1							i						
[651]		1	_		+	1	_	3		7		1	+	2		4		1											
[732]	+	25	_	4		23	+	1		7	+	8		1	_	4		0		_	5	+	5	_	1		2		1
[741]	+	29	_	6	_	4		5	_	17	+	3	+	3	+	9	_	3		-	5	+	1	+	5	_	1	_	3
$[82^2]$	+	10	+	14	+	18	_	1		12	_	18	+	6	+	9		0		-	2	_	12		0	+	12		6
[831]	+	23	+	23	+	52	_	3		27	_	5	_	11	_	5	+	5		-	4	-	27	+	5	+	5	+	10
[921]	+	17	+	19	+	37	+	23	+	73	+	39	_	6	_	30	_	7		-	3	-	20	_	22	_	54	+	13
$[101^{2}]$	1	- 1				ſ		- 1	+	ſ		- 1		1		1		- 1		-	1	-	7	<u>-</u>	17	_	20	_	45
[3º]	1	- 1						- 1	_	i	-			ì				- 1		1							j		
[75]	l .	- 1						- 1				- 1		- 1		- 1			_	1					- 1				
[84]	1	- 1		J.				- 1		l i		1		- 1		- 1		- 1	+	- 1			- 1		i				1
[93]	1	- 1		- 1		- 1		- 1		- 1		- 1		- 1		í		1	_	- 1			- 1	i	- 1		- 1		- 1
[102]	1	- 1		- 1				- 1		- 1						- 1			+	- 1		l .	- 1				- 1		
[111]		- 1		- 1						- 1										1		1	- 1	i	- 1		- 1		- 1
[12]	+	60	+	60	+:	120	+	30	+ 2	240	+:	120	+	40	+:	180	+	60	+	2 -	12		72	-	72	1	80	1	80

Table II. (lower half continued): Roots in Terms of Coefficients.

 $(32^{8}1^{3}) \ (2^{5}1^{2}) \ (51^{7}) \ (421^{6}) \ (321^{6}) \ (32^{2}1^{5}) \ (2^{4}1^{4}) \ (41^{8}) \ (321^{7}) \ (2^{3}1^{6}) \ (31^{9}) \ (2^{2}1^{8}) \ (21^{10}) \ (1^{12})$ $[5321^{2}]$ $[541^{3}]$ $[62^{2}1^{2}]$ $[631^3]$ $[721^{3}]$ $[81^{4}]$ $[3^4]$ $[43^{2}2]$ $[4^22^2]$ $[4^{2}31]$ $[532^{2}]$ $[53^{2}1]$ [5421] $[5^21^2]$ $[62^{3}]$ [6321] $[641^{2}]$ $[72^21]$ $[731^{2}]$ $[821^{2}]$ [91] $[4^3]$ [543] $[5^{2}2]$ $[63^{2}]$ [642][651][732][741] $[82^{2}]$ [831] 3 + [921] 23 $[101^{2}]$ + 27 - 30 + $[6^2]$ [75] [84] [93] - 21 3 18 21 [102] 17 32 + 202 16 [111] 1 + 35 10 +130 +87 19 [12] |12| + |84| + |42| + |252| + |105|